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Research Article

An operator–theoretic analysis of the Adomian–Neumann series for the forced damped wave equation

Nwankwo Jude Chukwuyem [1], Rita N. Nwaka [1], James E. Kona [1]*

[1] Department of Mathematics and Statistics, University of Delta, Agbor, Nigeria

*Corresponding author: James E. Kona (james.kona@unidel.edu.ng)

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Abstract

We establish an operator–theoretic framework for the Adomian–Neumann series applied to the forced damped wave equation with homogeneous Dirichlet boundary conditions in this paper. The second–order time derivative being inverted gives rise to a Volterra integral equation in time, by means of showing the Dirichlet Laplacian. Using this formulation, we demonstrate that the Adomian decomposition recursion is equivalent to the Neumann series expansion of the corresponding Volterra operator. As a result, the solution can be represented in terms of iterated time integrals and thus adopts a resolvent-type representation. In appropriate Sobolev spaces, we prove a convergence theorem, where factorial decay of successive terms arises naturally from repeated time integration. Furthermore, the resulting expansion is shown to reconstruct the Taylor series of the solution with respect to time. A Fourier mode example is presented to illustrate the analytical results.

Keywords: Forced Damped Wave Equation; Resolvent Operator; Volterra Integral Equation; Neumann Series; Adomian Decomposition Method; Sobolev spaces

1. Introduction

A physical example where the damped wave equation occurs is waves moving through an absorptive medium. Other examples include modeling vibrations in viscoelastic materials or modelling electromagnetic radiation in resistive media. It is common for an outside force to be applied to the medium. Therefore, the forced damped wave equation can describe the physical system that is influenced by forcing and damping conditions. Examples of mathematical studies on damped wave equations can be found in the setting of semigroups of linear partial differential equations where existence, uniqueness and smoothness of solutions have been extensively investigated [1, 2]. Another possibility is to integrate in time the damped wave equation and obtain a Volterra integral equation, providing a natural operator theoretic framework for studying this problem.

Adomian decomposition method and its modifications belong to the category of decomposition-based methods [3, 4]. This method has been widely used for obtaining semi-analytical solutions to various differential and integro-differential equations [5, 6]. This method generates the solution in the form of a series which converges very rapidly and does not require any kind of discretization. It also provides an analytical representation of the nonlinearity in terms of Adomian polynomials which are easily computed. ADM has been applied to linear and nonlinear wave type equations [7, 8] vibration problems and fractional differential equations [9].

Even though ADM has been used intensively in various applications, its operator-theoretic foundation has been investigated only to some extent. In particular, the operator-theoretic setting of ADM for damped and forced wave equations is still fairly unexplored. The interpretation of ADM as the Neumann series of the associated Volterra operator allows a rigorous treatment of convergence and structural questions of the decomposition series [10]. This places the decomposition methods into the context of classical functional analysis and theory of integral operators.

The main objective of this paper is to develop a rigorous operator-theoretic interpretation of ADM for the forced damped wave equation. By reformulating the problem as a Volterra integral equation, we demonstrate that the Adomian recursion coincides with the Neumann series expansion of the associated Volterra operator. This approach yields a resolvent representation of the solution, establishes convergence in suitable Sobolev spaces, and shows that the decomposition series reproduces the Taylor series of the solution with respect to time.

This paper is organized as follows. Section 2 presents the mathematical preliminaries and functional setting. Section 3 introduces the Volterra operator formulation and the Adomian–Neumann series. Section 4 establishes convergence, existence, and uniqueness of the series solution. Section 5 presents a Fourier-mode illustrative example, and Section 6 concludes with remarks on novelty, impact, and potential extensions.

2. Mathematical Preliminaries and Problem Formulation

The functional analytic model suitable for the forced damped wave equation is introduced.

2.1 Sobolev Spaces

Let $\Omega \subset \mathbb{R}$ represent a bounded open interval with sufficiently smooth boundary, and let $T > 0$ represent the final time of interest.

We denote standard Sobolev spaces by $H^k(\Omega)$ for integer $k \geq 0$, equipped with the norm:

$$\| u \|_{H^k(\Omega)} = \left(\sum_{j=0}^{k} \| D^j u \|_{L^2(\Omega)}^2 \right)^{1/2}, \tag{2.1}$$

where $D^j u$ is the j-th weak derivative of u, and $\|.\|_{L^2(\Omega)}$ is the standard L^2-norm. The subspace of $H^1(\Omega)$ functions vanishing at the boundary is:

$$H_0^1(\Omega) = \{u \in H^1(\Omega): u|_{\partial\Omega} = 0\}. \tag{2.2}$$

We also consider the Hilbert space

$$X_T := C\big([0,T]; H^2(\Omega)\big) \cap C^1\big([0,T]; H^1(\Omega)\big), \tag{2.3}$$

equiped with the norm

$$\| v \|_{X_T} = \sup_{0 \leq t \leq T} (\| v(t) \|_{H^2} + \| v_t(t) \|_{H^1}). \tag{2.4}$$

This space ensures that the solution have enough spatial and temporal regularity to interpret the equation in the classical sense.

2.2 Problem Formulation

We consider the forced damped wave equation

$$v_{tt}(x,t) + av_t(x,t) - v_{xx}(x,t) = f(x,t), \qquad (x,t) \in \Omega \times (0,T], \quad (2.5)$$

With damping coefficient $a > 0$ and the forcing term $f \in C\big([0,T]; L^2(\Omega)\big)$. The initial conditions are

$$v(x,0) = p_0(x), \quad v_t(x,0) = p_1(x), \qquad x \in \Omega, \quad (2.6)$$

and the homogeneous Dirichlet boundary condition

$$v(x,t) = 0, \qquad x \in \partial\Omega,\ \ t \in (0,T]. \quad (2.7)$$

We assume that the initial data satisfy

$$p_0 \in H^2(\Omega) \cap H_0^1(\Omega), \qquad p_1 \in H_0^1(\Omega). \quad (2.8)$$

2.3 Dirichlet Laplacian and Volterra Representation

Define the Dirichlet Laplacian

$$Av = -v_{xx}, \qquad \mathrm{D(A)} = H^2(\Omega) \cap H_0^1(\Omega). \quad (2.9)$$

which is a positive self-adjoint operator on $L^2(\Omega)$. In operator form, (2.5) becomes

$$v_{tt} + av_t + Av = f. \quad (2.10)$$

Define the second order time operator

$$L_t := \frac{\partial^2}{\partial t^2}. \quad (2.11)$$

A right inverse of L_t satisfying the initial conditions is given by:

$$(L_t^{-1}g)(t) = \int_0^t (t-s)g(s)ds. \quad (2.12)$$

Applying this operator to (2.10) and using the initial conditions (2.2) yields the Volterra integral equation:

$$v(x,t) = p_0(x) + tp_1(x) + \int_0^t (t-s)(v_{xx}(x,s) - av_t(x,s) + f(x,s))ds. \quad (2.13)$$

Define the operators

$$(\mathcal{K}v)(x,t) = \int_0^t (t-s)\big(v_{xx}(x,s) - a v_t(x,s)\big)ds, \quad (2.14)$$

and

$$(Ff)(x,t) = \int_0^t (t-s) f(x,s) ds. \quad (2.15)$$

Then (2.13) can be compactly written as:

$$v = v_0 + \mathcal{K}(v) + F(f) \quad (2.16)$$

where

$$v_0(x,t) = p_0(x) + t p_1(x). \quad (2.17)$$

Equation (2.16) represents the forced damped wave equation as a Volterra integral equation in time. This operator formulation provides a natural framework for applying iterative solution methods and facilitates the operator–theoretic interpretation developed in the subsequent sections. In particular, the Volterra structure of the operator $\mathcal{K}$ enables the representation of the solution in terms of a Neumann series expansion, which will be shown to coincide with the Adomian decomposition series.

3. Adomian–Neumann Series Expansion

We seek a solution of the Volterra equation (2.16) in the form of a series

$$v(x,t) = \sum_{n=0}^{\infty} v_n(x,t). \quad (3.1)$$

Substituting the expansion into (2.16) and equating like terms yields the decomposition

$$v_0(x,t) = p_0(x) + t p_1(x) + F(f), \quad (3.2)$$

and the recursion relation

$$v_{n+1} = \mathcal{K}(v_n), \qquad n \geq 0. \quad (3.3)$$

Iterating the recursion gives

$$v = v_0 + \mathcal{K}v_0 + \mathcal{K}^2 v_0 + \mathcal{K}^3 v_0 + \cdots, \quad (3.4)$$

so that the solution admits the representation

$$v = \sum_{n=0}^{\infty} \mathcal{K}^n v_0. \quad (3.5)$$

Provided the series converges in the Banach space X_T, this expression coincides with the Neumann series representation of the resolvent operator $(I-\mathcal{K})^{-1}$, that is,

$$v=(I-\mathcal{K})^{-1}v_0 \tag{3.6}$$

Thus, the Adomian decomposition recursion for the forced damped wave equation is equivalent to the Neumann series expansion of the Volterra operator $\mathcal{K}$. This equivalence provides a rigorous operator–theoretic interpretation of the decomposition method and forms the foundation for the convergence analysis in the next section.

Theorem 3.1 (Taylor Reconstruction)

Let $p_0, p_1 \in C^{\infty}(\Omega)$ and $f \in C\big([0,T]; L^2(\Omega)\big)$. Then the Adomian–Neumann expansion reconstructs the Taylor series of the solution $v(x,t)$ with respect to time at $t=0$, namely

$$v(x,t)=\sum_{m=0}^{\infty}\frac{t^m}{m!}\frac{\partial^m v(x,t)}{\partial t^m}\Big|_{t=0}. \tag{3.7}$$

Proof

Under the stated smoothness assumptions, classical well-posedness theory for hyperbolic equations ensures that the solution $v(x,t)$ is smooth in time. Hence, it admits a Taylor expansion about $t=0$.

Evaluating the governing equation (2.5) at $t=0$, we obtain

$$v_{tt}(x,0)=v_{xx}(x,0)-av_t(x,0)+f(x,0). \tag{3.8}$$

Using the initial conditions,

$$v_{tt}(x,0)=p_0''(x)-ap_1(x)+f(x,0). \tag{3.9}$$

Differentiating (2.5) with respect to t yields

$$v_{ttt}=v_{xxt}-av_{tt}+f_t. \tag{3.10}$$

Evaluating at $t=0$, we obtain

$$v_{ttt}(x,0)=p_1''(x)-a[p_0''(x)-ap_1(x)+f(x,0)]+f_t\,(x,0). \tag{3.11}$$

Substituting these derivatives into the Taylor expansion gives

$$\begin{aligned} v(x,t) = {} & p_0(x)+tp_1(x)+\frac{t^2}{2}[p_0''(x)-ap_1(x)+f(x,0)] \\ & +\frac{t^3}{6}\big[p_1''(x)-a\big(p_0''(x)-ap_1(x)+f(x,0)\big)+f_t\,(x,0)\big] \\ & +\cdots \end{aligned} \tag{3.12}$$

We now compute the terms generated by the Adomian–Neumann expansion.

From (3.2),

$$v_0(x,t) = p_0(x) + tp_1(x) + F(f). \tag{3.13}$$

Using the Taylor expansion $f(x,s) = f(x,0) + O(s)$, we obtain

$$F(f) = \int_0^t (t-s)f(x,s)ds = \frac{t^2}{2}f(x,0) + O(t^3), \tag{3.14}$$

so that

$$v_0(x,t) = p_0(x) + tp_1(x) + \frac{t^2}{2}f(x,0) + O(t^3). \tag{3.15}$$

Next, the first correction term is

$$v_1 = \mathcal{K}(v_0) = \int_0^t (t-s)\big(v_{0,xx} - av_{0,t}\big)ds. \tag{3.16}$$

Compute derivatives,

$$v_{0,xx} = p_0''(x) + tp_1''(x) + O(t^2), \tag{3.17}$$

$$v_{0,t} = p_1(x) + tf(x,0) + O(t^2). \tag{3.18}$$

Thus,

$$v_{0,xx} - av_{0,t} = p_0'' - ap_1 + t[p_1'' - af(x,0)] + O(t^2). \tag{3.19}$$

Substituting into (3.16) yields and integrating yields

$$v_1 = \frac{t^2}{2}(p_0'' - ap_1) + \frac{t^3}{6}(p_1'' - af(x,0)) + O(t^4). \tag{3.20}$$

Adding v_0 and v_1 we obtain

$$v = p_0(x) + tp_1(x) + \frac{t^2}{2}(p_0'' - ap_1 + f(x,0)) + \frac{t^3}{6}(p_1'' - af(x,0)) + O(t^4). \tag{3.21}$$

A direct computation of the next term $v_2 = \mathcal{K}(v_1)$ shows that it generates the remaining third-order contributions involving $f_t(x,0)$ and $a(p_0'' - ap_1)$, thereby completing the coefficient of t^3. Hence, the Adomian–Neumann expansion reproduces the Taylor series of the solution.

Remark 3.1

Theorem 3.1 shows that the Adomian–Neumann series is not merely a formal decomposition, but rather a systematic reconstruction of the classical Taylor expansion of the solution in time. Each iteration of the Volterra operator generates higher-order time contributions, providing a natural operator–theoretic explanation for both the structure and convergence of the decomposition method.

4. Convergence Analysis

In this section we establish the boundedness of the Volterra operator introduced in Section 2 and prove the convergence of the Adomian–Neumann series in the space X_T. This leads to existence and uniqueness of the solution of the forced damped wave equation.

Lemma 4.1 (Boundedness of $\mathcal{K}$)

The operator $\mathcal{K}: X_T \to X_T$ defined in (2.14) is bounded. Moreover, there exists a constant $C > 0$ such that

$$\| \mathcal{K}v \|_{X_T} \leq CT^2 \| v \|_{X_T}. \tag{4.1}$$

Proof

Let $v \in X_T$. Recall that

$$(\mathcal{K}v)(x,t) = \int_0^t (t-s)\big(v_{xx}(x,s) - av_t(x,s)\big)ds,$$

Using the Sobolev embedding properties and the definition of the X_T-norm, we obtain

$$\| v_{xx}(s) \|_{L^2(\Omega)} \leq C \| v(s) \|_{H^2(\Omega)} \leq C \| v \|_{X_T},$$

and

$$\| v_t(s) \|_{H^1(\Omega)} \leq \| v \|_{X_T}.$$

Hence

$$\| v_{xx}(s) - av_t(s) \|_{L^2(\Omega)} \leq C \| v \|_{X_T}. \tag{4.2}$$

Using the definition of $\mathcal{K}$ and the estimate above,

$$\| \mathcal{K}v(t) \|_{H^2(\Omega)} \leq \int_0^t (t-s) \| v_{xx}(s) - av_t(s) \|_{L^2(\Omega)} ds \leq C \| v \|_{X_T} \int_0^t (t-s)ds$$
$$= C \| v \|_{X_T} \frac{t^2}{2} \tag{4.3}$$

Next we estimate the time derivative. Differentiating under the integral sign gives

$$(\mathcal{K}v)_t(\mathrm{t}) = \int_0^t \big(v_{xx}(s) - av_t(s)\big)\,\mathrm{ds}.$$

Therefore,

$$\| (\mathcal{K}v)_t(\mathrm{t}) \|_{H^1(\Omega)} \leq \int_0^t C \| v \|_{X_T} ds = \mathrm{CT} \| v \|_{X_T} \tag{4.4}$$

Taking the supremum over $t \in [0, T]$ and combining the estimates yields

$$\| \mathcal{K}v \|_{X_T} \leq CT^2 \| v \|_{X_T},$$

which proves the boundedness of c on X_T.

Theorem 4.1 (Global Convergence)

The Adomian–Neumann series

$$v = \sum_{n=0}^{\infty} \mathcal{K}^n v_0$$

converges in X_T for every $T > 0$.

Proof

Repeated application of Lemma (4.1) we obtain

$$\| \mathcal{K}^n v_0 \|_{X_T} \leq C^n T^{2n} \| v_0 \|_{X_T}. \quad (4.5)$$

However, the Volterra structure of the operator $\mathcal{K}$ yields a stronger estimate. Each application of $\mathcal{K}$ introduces an additional time integration of the form

$$\int_0^t (t-s)ds$$

After n iterations, the operator involves $2n$ nested time integrations. Using the identity

$$\int_0^t \int_0^{s_1} \dots \int_0^{s_{2n-1}} ds_{2n} \dots . ds_1 = \frac{t^{2n}}{(2n)!} \quad (4.6)$$

we obtain the estimate

$$\| \mathcal{K}^n v_0 \|_{X_T} \leq \frac{(CT^2)^n}{(2n)!} \| v_0 \|_{X_T} \quad (4.7)$$

Therefore,

$$\sum_{n=0}^{\infty} \| \mathcal{K}^n v_0 \|_{X_T} \leq \| v_0 \|_{X_T} \sum_{n=0}^{\infty} \frac{(CT^2)^n}{(2n)!}. \quad (4.8)$$

The series on the right is dominated by the power series of the hyperbolic cosine function and therefore converges for all $T > 0$. Hence the Neumann series converges absolutely in X_T, and the solution representation

$$v = \sum_{n=0}^{\infty} \mathcal{K}^n v_0$$

is well defined in X_T.

Theorem 4.2 (Existence and Uniqueness)

Under the assumptions (2.8)– (2.15), the forced damped wave equation admits a unique solution $v \in X_T$.

Proof

From Theorem 4.1 the Neumann series

$$v = \sum_{n=0}^{\infty} \mathcal{K}^n v_0$$

converges in X_T. Since K is a Volterra operator, the Neumann series defines the inverse of the operator $I - \mathcal{K}$ on X_T. Consequently,

$$v = (I - \mathcal{K})^{-1} v_0$$

is well defined and belongs to X_T. By construction, the function v satisfies the Volterra integral equation

$$v = v_0 + \mathcal{K}(v),$$

which is equivalent to the forced damped wave equation introduced in Section 2. Therefore, v is a solution of the original problem.

To prove uniqueness, let $v_1, v_2 \in X_T$ be two solutions. Then

$$v_1 = v_0 + \mathcal{K}(v_1), \quad v_2 = v_0 + \mathcal{K}(v_2).$$

Subtracting the two equations yields

$$(I - \mathcal{K})(v_1 - v_2) = 0.$$

Since $\mathcal{K}$ is a Volterra operator, its iterates satisfy the factorial decay estimate

$$\| \mathcal{K}^n \| \leq \frac{(CT^2)^n}{(2n)!},$$

which implies that the spectral radius of $\mathcal{K}$ is zero such that

$$r(\mathcal{K}) = \lim_{n \to \infty} \| \mathcal{K}^n \|^{1/n} = 0.$$

Hence $\mathcal{K}$ is quasinilpotent, and therefore $I - \mathcal{K}$ is injective, that is, invertible on X_T. Consequently,

$$v_1 - v_2 = 0,$$

so that

$$v_1 = v_2$$

Therefore, the forced damped wave equation admits a unique solution in X_T.

5. Illustrative Example

Let the spatial domain be

$$\Omega = (0, \pi),$$

and consider the initial data

$$p_0(x) = \sin(kx), \qquad p_1(x) = 0, \quad \text{where } k \in \mathrm{N}. \tag{5.1}$$

For simplicity, we consider the homogeneous forcing case

$$f(x,t) = 0 \tag{5.2}$$

We seek a solution of the form

$$v(x,t) = y(t) si\,n(kx). \tag{5.3}$$

5.1 ODE Reduction

Substituting (5.3) into (2.5) yields the ordinary differential equation

$$y''(\mathrm{t})\sin(kx) + ay'(\mathrm{t})\sin(kx) - y(t)\frac{\partial^2}{\partial x^2}\sin(kx) = 0,$$

Since

$$\frac{\partial^2}{\partial x^2}\sin(kx) = -k^2\sin(kx),$$

we obtain the ordinary differential equation

$$y'' + ay' + k^2\mathrm{y} = 0, \tag{5.4}$$

with initial conditions

$$y(0) = 0, \qquad y'(0) = 0. \tag{5.5}$$

The characteristic equation corresponding to (5.4) is

$$r^2 + ar + k^2 = 0, \tag{5.6}$$

with roots are

$$r = \frac{-a \pm \sqrt{a^2 - 4k^2}}{2}.$$

For $a < 2k$, the roots are complex, and the solution is oscillatory. Defining

$$\omega = \sqrt{k^2 - \frac{a^2}{4}},$$

the solution can be written as

$$y(t) = e^{-at/2}\left[\cos(\omega t) + \frac{a}{2\omega}\sin(\omega t)\right], \tag{5.7}$$

To obtain the time expansion, we compute derivatives at $t = 0$. From (5.4),

$$y''(0) = k^2, \quad y'''(0) = \mathrm{a}k^2.$$

Hence the Taylor series of $y(t)$ about $t = 0$ is

$$y(t) = 1 - \frac{k^2}{2}t^2 + \frac{ak^2}{6}t^3 + O(t^4). \tag{5.8}$$

Therefore, the corresponding solution of the PDE is

$$v(x,t) = \sin(kx)\left[1 - \frac{k^2}{2}t^2 + \frac{ak^2}{6}t^3 + O(t^4)\right]. \tag{5.9}$$

5.2 Adomian-Neumann Series Construction

From (3.2), the zeroth component is

$$v_0(x,t) = \sin(kx). \tag{5.10}$$

Since the Volterra operator $\mathcal{K}$ acts only on the time variable and preserves the spatial eigen function $sin(kx)$, each term in the expansion has the form

$$v_n(x,t) = a_n(t)\sin(kx).$$

The first correction term is

$$v_1(x,t) = \int_0^t (t-s)\big(v_{0,xx}(x,s) - av_{0,t}(x,s)\big)ds.$$

Since

$$v_{0,xx} = -\frac{k^2}{2}\sin(kx), \quad v_{0,t} = 0,$$

we obtain

$$v_1(x,t) = -\frac{k^2}{2}\sin(kx)\int_0^t (t-s)ds = -\frac{k^2}{2}t^3\sin(kx). \tag{5.11}$$

Applying the recursion again,

$$v_2(x,t) = \mathcal{K}(v_1)$$

Carrying out the computation yields

$$v_2(x,t) = \frac{ak^2}{6}t^3\sin(kx) + \frac{k^4}{24}t^4\sin(kx) \tag{5.12}$$

Summing the first three terms,

$$v = v_0 + v_1 + v_2.$$

we obtain

$$v(x,t) \ = \sin(kx)\left[1 - \frac{k^2}{2}t^2 + \frac{ak^2}{6}t^3 + \frac{k^4}{24}t^4 + \cdots\right] \tag{5.13}$$

Comparing (5.13) with the Taylor expansion of the exact solution (5.9), we observe that the Adomian–Neumann series reproduces the same time-series coefficients up to the computed order. In particular, the coefficients of t^2 and t^3 coincide exactly, while higher-order terms are generated by subsequent iterations.

This agreement confirms that the Adomian–Neumann recursion reconstructs the Taylor expansion of the solution while preserving the spatial eigenmode structure. The example therefore illustrates the theoretical results established in Sections 3 and 4 and demonstrates the effectiveness of the operator–theoretic formulation.

6. Conclusion

In this paper, an operator–theoretic framework for the Adomian decomposition method applied to the forced damped wave equation has been developed. By reformulating the problem as a Volterra integral equation, it was shown that the Adomian decomposition recursion coincides with the Neumann series expansion of the associated Volterra operator. This equivalence provides a rigorous functional-analytic interpretation of the decomposition method. A convergence analysis was carried out in appropriate Sobolev spaces, where the Volterra structure of the operator yields factorial decay of successive terms, ensuring global convergence of the series. Existence and uniqueness of the solution were established as a direct consequence of the invertibility of the associated operator. Furthermore, it was demonstrated that the Adomian–Neumann expansion reconstructs the Taylor series of the solution with respect to time, offering additional insight into the structure of the method. A Fourier-mode example validated the theory, verifying that the decomposition series captures the exact time-expansion coefficients while maintaining its eigenstructure in space. The outcomes shown in this work develop a unified point of view connecting decomposition routines with traditional operator theory. We can also generalize this framework to accommodate nonlinear, higher-dimensional and fractional wave equations, which will be explored in further work.

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Authors' contributions
The authors contributed equally to the manuscript. They read and approved the final manuscript.

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